NAME _____ DATE _____

4–2 Adding and Subtracting Polynomials

Objective: To add and subtract polynomials.

Vocabulary

Monomial An expression that is either a numeral, a variable, or the product of a numeral and one or more variables. For example: 13, m, $8c$, $2xy$, $5p^2q$.

Coefficient In the monomial $9x^2y^3$, 9 is the coefficient, or numerical coefficient.

Similar, or like, terms Two monomials that are exactly alike or the same except for their numerical coefficient. For example, $-3xy$ and $7xy$ are similar.

Constant monomial or constant A numeral such as 7.

Polynomial A sum of monomials. For example, $x^2 + 3x + y^2 + 2$.

Binomial A polynomial of only two terms. For example, $2x + 5$.

Trinomial A polynomial of only three terms. For example, $a^2 + 2ab + b^2$.

Simplified form, or simplest form, of a polynomial A polynomial which has no two of its terms similar.

CAUTION When a monomial does not have a written numerical coefficient, remember that its coefficient is 1. For example, $x^6y^2 = 1x^6y^2$.

Example 1 Simplify $-5x^3 + 2x^2 + x^2 + 7x^3 - 4$.

Solution
$$-5x^3 + 2x^2 + x^2 + 7x^3 - 4 = (-5x^3 + 7x^3) + (2x^2 + x^2) - 4$$
$$= (-5 + 7)x^3 + (2 + 1)x^2 - 4$$
$$= 2x^3 + 3x^2 - 4$$

Simplify.

1. $2x - y + 3x - 2y$ **$5x - 3y$**
2. $7m - 5n - 2m + n$ **$5m - 4n$**
3. $4x^2 - 3x - 2x^2 + 7x - 2$ **$2x^2 + 4x - 2$**
4. $n^2 - 3n - 5n^2 + 7n + 6n^2$ **$2n^2 + 4n$**
5. $a^2 + 2ab - 5ab + 4a^2$ **$5a^2 - 3ab$**
6. $x^2y - y^3 - 8x^2y + 5y^3$ **$-7x^2y + 4y^3$**
7. $a^2b - 2ab^2 + 5a^3 - 3a^2b$ **$5a^3 - 2a^2b - 2ab^2$**
8. $-6xy^2 + 5x^2y - x^3 + xy^2 + 3x^3 - 2x^2y$ **$2x^3 + 3x^2y - 5xy^2$**

Example 2 Add $2x^2 + 5xy - 6y^2$ and $8x^2 + 6xy + y^2$.

Solution 1 First group similar terms and then combine them.
$$(2x^2 + 5xy - 6y^2) + (8x^2 + 6xy + y^2) = (2x^2 + 8x^2) + (5xy + 6xy) + (-6y^2 + y^2)$$
$$= 10x^2 + 11xy - 5y^2$$

Solution 2
$$\begin{array}{r} 2x^2 + 5xy - 6y^2 \\ 8x^2 + 6xy + y^2 \\ \hline 10x^2 + 11xy - 5y^2 \end{array}$$
You can also align similar terms vertically and add.

4–2 Adding and Subtracting Polynomials *(continued)*

Vocabulary

Degree of a variable in a monomial The number of times that the variable occurs as a factor in a monomial. For example, in $7x^3y$, the degree of x is 3, and the degree of y is 1.

Degree of a monomial The sum of the degrees of its variables. For example, the degree of $8x^2y^3$ is 5. The degree of any nonzero constant monomial, such as 10, is 0.

Degree of a polynomial The greatest of the degrees of its terms after it has been simplified. For example, the degree of $-5x^3 + 2x^2 + x^2 + 5x^3 - 4$ is 2, since it can be simplified to $3x^2 - 4$.

Add.

9. $\begin{array}{r} 3a - 1 \\ 4a + 3 \\ \hline \end{array}$ **$7a + 2$**
10. $\begin{array}{r} 4n + 2 \\ -2n - 5 \\ \hline \end{array}$ **$2n - 3$**
11. $\begin{array}{r} 2x - 3y \\ -2x + 6y \\ \hline \end{array}$ **$3y$**
12. $\begin{array}{r} 5n - 2p \\ -3n + 5p \\ \hline \end{array}$ **$2n + 3p$**
13. $\begin{array}{r} 4x - 5y + 3 \\ -2x + 7y + 7 \\ \hline \end{array}$ **$2x + 2y + 10$**
14. $\begin{array}{r} 2a - 3b - 6 \\ 3a - b + 8 \\ \hline \end{array}$ **$5a - 4b + 2$**
15. $\begin{array}{r} 6x^2 - 3x + 2 \\ 2x^2 + x - 5 \\ \hline \end{array}$ **$8x^2 - 2x - 3$**
16. $\begin{array}{r} 5 - 2n - 6n^2 \\ -3 + n - 2n^2 \\ \hline \end{array}$ **$2 - n - 8n^2$**
17. $\begin{array}{r} 4c^2 - 3cd - 5d^2 \\ -c^2 + 6cd - 2d^2 \\ \hline \end{array}$ **$3c^2 + 3cd - 7d^2$**
18. $\begin{array}{r} 6a^2 - 2ab \\ -2a^2 + 5ab - b^2 \\ \hline \end{array}$ **$4a^2 + 3ab - b^2$**
19. $\begin{array}{r} 3x - 2y - 5z + 1 \\ 2x + y - 3z \\ 3y + z + 3 \\ \hline \end{array}$ **$5x + 2y - 7z + 4$**
20. $\begin{array}{r} 6a - 2b + 4 \\ 3a - 5c - 1 \\ -a - b + 6c + 5 \\ \hline \end{array}$ **$8a - 3b + c + 8$**

Example 3 Subtract $-x^2 + 5xy + 6y^2 - 3$ from $3x^2 - 6xy - 2y^2 - 5$.

Solution 1 Add the opposite of $(-x^2 + 5xy + 6y^2 - 3)$ to $3x^2 - 6xy - 2y^2 - 5$.
$$(3x^2 - 6xy - 2y^2 - 5) - (-x^2 + 5xy + 6y^2 - 3) =$$
$$(3x^2 - 6xy - 2y^2 - 5) + (x^2 - 5xy - 6y^2 + 3) = 4x^2 - 11xy - 8y^2 - 2$$

Solution 2 You can also align similar terms vertically.
$$\begin{array}{r} 3x^2 - 6xy - 2y^2 - 5 \\ -(-x^2 + 5xy + 6y^2 - 3) \\ \hline \end{array} \longrightarrow \text{Change to the opposite signs and add.} \longrightarrow \begin{array}{r} 3x^2 - 6xy - 2y^2 - 5 \\ x^2 - 5xy - 6y^2 + 3 \\ \hline 4x^2 - 11xy - 8y^2 - 2 \end{array}$$

21–30. In Exercises 9–18, subtract the lower polynomial from the upper one.

21. $-a - 4$ 22. $6n + 7$ 23. $4x - 9y$ 24. $8n - 7p$
25. $6x - 12y - 4$ 26. $-a - 2b - 14$ 27. $4x^2 - 4x + 7$
28. $8 - 3n - 4n^2$ 29. $5c^2 - 9cd - 3d^2$
30. $8a^2 - 7ab + b^2$

Mixed Review Exercises

Simplify.

1. -2^3 **-8**
2. $(-3)^2$ **9**
3. $2^2 + 3^2$ **13**
4. $(2 + 3)^2$ **25**

Solve.

5. $3(y + 2) - 2 = 2(4 - y)$ $\left\{\frac{4}{5}\right\}$
6. $10 = 2(n + 3)$ **{2}**
7. $4(x - 10) = 13 - 3(2x + 1)$ **{5}**
8. $-\frac{2}{5}(n + 3) = 10$ **{−28}**
9. $c - 2 = |1 - 8|$ **{9}**
10. $\frac{3}{4}(2y - 6) = y - 7$ **{−5}**